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| December 7, 2013 | Matt Landreman |

# Transport matrix in SFINCS and Beidler et al (2011)

In these notes we first derive the relationships between the transport matrix elements in the SFINCS single-species documentation and the matrix elements defined in eq (4) of Beidler et al NF (2011). Next, we show that in axisymmetry and the limit of high collisionality, the transport matrices given in the two references agree. Since Beidler uses SI units whereas the SFINCS documentation uses Gaussian units, we replace  everywhere in the SFINCS formulae to convert them to SI, which means we also replace , , and .

## Relating sfincs transport matrix elements to Beidler’s notation

Examining (4) in Beidler and the equations that follow it, we can schematically write each element of the  transport matrix by setting 2 of the 3 thermodynamic forces to 0, giving

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and

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The superscript  has been added to  to distinguish Beidler’s transport matrix  from the SFINCS transport matrix . Beidler says “ and  are reference values of the torus major radius and magnetic field strength, respectively.” In SFINCS,  is defined specifically as the  Fourier harmonic of the Boozer spectrum. It seems safe to take Beidler’s  to have this same definition.

For comparison, the sfincs definitions are

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and

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Comparing the 2 sets of definitions, then, we find

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The relations - may be useful for relating the transport matrix output by SFINCS to DKES or other codes. Notice and have the same sign, whereas and have opposite sign, as do and .

## Beidler’s energy-integrated matrix elements for axisymmetry

From p13 of Beidler, at large collisionality the monoenergetic coefficients are

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and

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where

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Here,  where . This definition of the pitch-angle scattering frequency agrees with (3.45) in Per’s textbook.

On p3, Beidler says the energy-integrated transport matrix is related to the monoenergetic coefficients by



where  and . Switching the integration variable to the speed ,

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Computing the energy integrals of - gives







and



To compare these results to the SFINCS documentation, we must recall in SFINCS that

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Then using - in these notes, and using (155) in the SFINCS single-species documentation, we find the SFINCS predictions for Beidler’s matrix elements are

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It is now apparent that - equal - if make the following replacements, both of which seem reasonable:



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The replacement follows from equating 2 relations for the toroidal flux: , noting . Both (155) in the SFINCS documentation and Beidler agree that , , , and  are nearly 0. Thus, the SFINCS documentation and Beidler agree on all the ion transport matrix elements in axisymmetry at high collisionality.